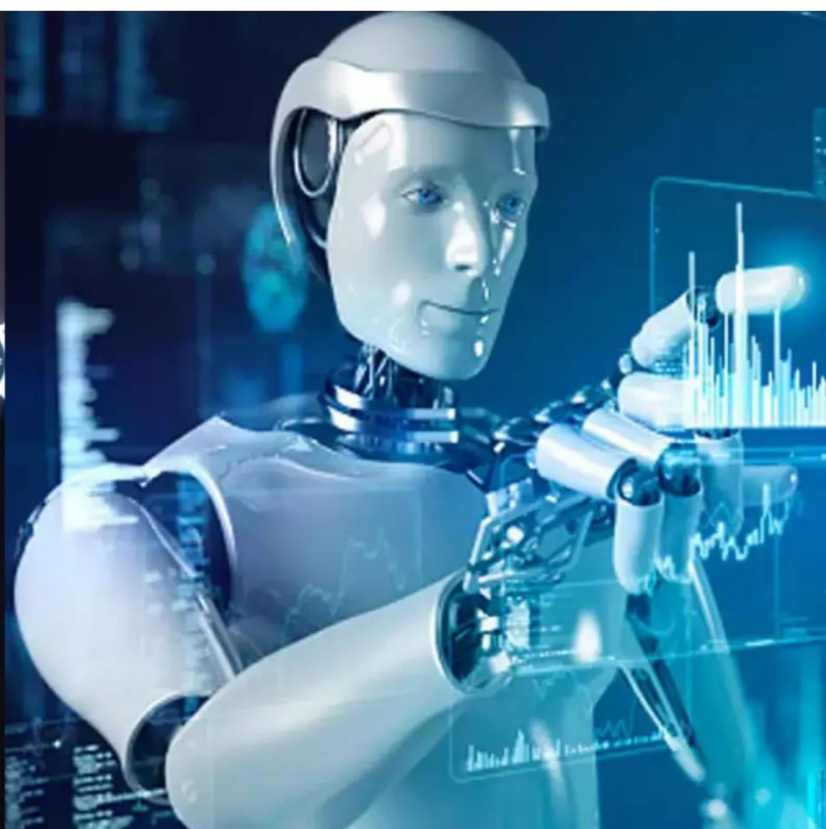




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Bianchi Type-V Cosmological Model for Perfect Fluid in $f(R)$ Theory of Gravity with Special Form of Deceleration Parameter

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ABSTRACT: We study the Bianchi type-V perfect-fluid model in $f(R)$ gravity. Using the correct field equations, we show that the assumed power-law form $F \propto a^n$, together with the special deceleration parameter $q(t) = -1 + (q_0 + 1)e^{-\alpha t}$, forces the model to be isotropic. The field equations further restrict the parameters. Two physically relevant solutions are obtained: a dust model with $\omega = 0$, $n = 1$, $\alpha^2 = 4b^2/3$, and a radiation model with $\omega = 1/3$, $n = 0$, $\alpha^2 = 2b^2$. The Ricci scalar and $f(R)$ are assessed.

KEYWORDS: $f(R)$ theory of gravity, Bianchi type-V model, Perfect fluid, Deceleration parameter.

I. INTRODUCTION

The gravitational events have been described by the general theory of relativity, but the nature of the negative component known as dark energy and the universe's rapid expansion remain unexplained.

We know from contemporary cosmic observational data that the expansion of our universe is accompanied by an acceleration. One alternate technique that results in modified theories of gravitation is to modify the Einstein–Hilbert action to explain such difficulties of current cosmic acceleration. Because of its significance for cosmology, the $f(R)$ theory of gravity is regarded as the most appropriate theory of gravity. The description of the universe's evolution relies heavily on the $f(R)$ theory of gravity. The dark energy option is provided by the $f(R)$ theory of gravity. The $f(R)$ gravity, which is a version of Albert Einstein's general theory of relativity, was initially presented in 1970 by Hans Adolph Buchdahl [1].

Many authors explained the phenomenon of $f(R)$ theory of gravity. The $f(R)$ theory of gravity has been researched by Valerio Faraoni and Thomas P. Sotiriou [2]. M. Farasat Shamir [3] has studied $f(R)$ gravity Bianchi type cosmological models.

In $f(R)$ gravity, M. Sharif and M. Farasat Shamir [4] have assessed non-vacuum Bianchi type I and V models. Anisotropic Bianchi type-III model in $f(R)$ gravity was explained by M. Sharif and H. Rizwana Kausar [5] on the assumptions of exponential and power-law volumetric expansion, which indicate an accelerated expansion of the universe. Excellent work on context has been done by several authors. M. Sharif and H. Rizwana Kausar [6] proposed non-vacuum solutions of Bianchi Type-V universe with $f(R)$ gravity using isotropic perfect fluid. Because the Bianchi models are anisotropic, they can be used to analyze the universe's isotropic behavior throughout time. The functional form of $f(R)$ with power-law expansion in the anisotropic model has been analyzed by Singh V. and Singh C.P. [7]. K.S. Adhav [8] has explained Bianchi type III cosmological model in $f(R)$ theory of gravity. In $f(R)$ gravity, the cosmic acceleration has been extensively investigated, and one important function that the scalar field can play in explaining it is [9].

By employing a unique sort of deceleration parameter, D.R.K. Reddy, K.S. Adhav, and S.L. Munde [10] were able to achieve vacuum solutions for the Bianchi type I and V models. In the $f(R)$ theory of gravity, L. S. Ladke [11] has produced a Bianchi type I (Kasner Form) cosmological model. By using perfect fluid and $f(R)$ gravity, Kazuharu

Bamba, Shin'ichi Nojiri, Sergei D. Odintsov, and Diego Saez-Gomez [12] have defined the inflationary universe and compared it with empirical data. The anisotropic cosmological model with cosmic string in the $f(R)$ theory of gravity has been seen by A.Y. Shaikh and S.D. Katore [13]. Once more, they [14] have created a Bianchi type VI₀ cosmological model in $f(R)$ gravity utilizing bulk viscous matter and the unique form of the average scale factor by limiting the deceleration parameter. In the $f(R)$ theory of gravity, L.S. Ladke, R. A. Hiwarkar, and V. K. Jaiswal [15] have examined the precise vacuum solution of the Bianchi type V space-time and have talked about the dynamics of the universe expansion using a few physical parameters. Further they [16,17] studied the non-vacuum perfect fluid solution of LRS Bianchi type V and behavior of bulk viscous Bianchi V cosmological model in the $f(R)$ theory of gravity respectively.

Since the 1960s, the Bianchi type models have been the subject of extensive research and are significant to theoretical cosmology. Since the space-time by definition admits a three parameter group of isometries whose orbits are space-like hypersurfaces, Bianchi cosmology describes a spatially uniform cosmos. Anisotropy in the rate of expansion can have an impact on or relate to several parts of the physical cosmos, which can be analyzed using these models. Because Bianchi type-V cosmological models allow arbitrarily modest anisotropy levels at any constant of cosmic time and are an anisotropic version of the open FRW model, they are of far greater interest. They are appropriate as universe models because of these characteristics. The Bianchi type-V model has been thoroughly examined by several authors viz. L.S. Ladke [18], Anil Kumar Yadav [19], C.B. Collins [20], etc.

In this paper, we examine the Bianchi type-V non-vacuum solution within the $f(R)$ theory of gravity when a perfect fluid is present. The main goal is to establish a general solution of the field equations by making an assumption about a certain type of deceleration parameter. The physical parameters are discussed. Also examined are $f(R)$ and the Ricci scalar.

II. $F(R)$ THEORY OF GRAVITATION

A set of theories known as $f(R)$ gravity is characterized by distinct functions of the Ricci scalar. The action for $f(R)$ theory of gravity is given by

$$S = \int \sqrt{-g} \left[\frac{1}{16\pi G} f(R) + L_m \right] dx^4 \quad (1)$$

where $f(R)$ is a function of the Ricci scalar and L_m is the matter Lagrangian. Varying the action (1) with respect to the metric g_{ij} , we get the field equations in $f(R)$ gravity as

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \square F(R) = k T_{ij} \quad (2)$$

Where

$$F(R) = \frac{df(R)}{dR}, \quad \square \equiv \nabla^i \nabla_i$$

∇_i is the covariant derivative, T_{ij} is the standard matter energy-momentum tensor, and $k = 8\pi G/c^4$. Contracting (2) gives

$$F(R) R - 2 f(R) + 3 \square F(R) = k T \quad (3)$$

From (3),

$$f(R) = \frac{F(R)R}{2} + \frac{3}{2} \square F(R) - \frac{1}{2} k T \quad (4)$$

Substituting (4) into (2), we obtain the useful relation

$$FR_{ij} - \nabla_i \nabla_j F - k T_{ij} = \frac{1}{4} g_{ij} (F R - \square F - k T) \quad (5)$$

For diagonal metrics, define

$$K_i = \frac{FR_{ii} - \nabla_i \nabla_i F - k T_{ii}}{g_{ii}} \quad (6)$$

with no sum over i . Equation (5) implies that K_i is independent of the index i , hence $K_i - K_j = 0$ for all i, j .

III. BIANCHI TYPE-V MODEL

The line element of Bianchi type-V space-time is

$$ds^2 = dt^2 - A^2 dx^2 - e^{2bx} B^2 dy^2 - e^{2bx} C^2 dz^2 \quad (7)$$

where A, B, C are functions of t and b is a non-zero constant. The Ricci scalar for this metric is

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{3b^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \right] \quad (8)$$

The energy-momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \quad (9)$$

where $u_i u^i = 1$. Here ρ is the energy density and p is the pressure. The equation of state is

$$p = \omega \rho, \quad 0 \leq \omega \leq 1 \quad (10)$$

In a comoving coordinate system,

$$T_1^1 = T_2^2 = T_3^3 = -p, \quad T_{44} = \rho \quad (11)$$

The conservation equation $T_{;j}^i = 0$ gives

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (12)$$

Using (6), (9), and (11), the correct field equations for the metric (7) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{F}}{AF} + \frac{\ddot{F}}{F} + \frac{2b^2}{A^2} + \frac{k}{F}(\rho + p) = 0 \quad (13)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{B}\dot{F}}{BF} + \frac{\ddot{F}}{F} + \frac{2b^2}{A^2} + \frac{k}{F}(\rho + p) = 0 \quad (14)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{F}}{CF} + \frac{\ddot{F}}{F} + \frac{2b^2}{A^2} + \frac{k}{F}(\rho + p) = 0 \quad (15)$$

Subtracting (14) from (13), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) \left(\frac{\dot{C}}{C} + \frac{\dot{F}}{F} \right) = 0 \quad (16)$$

Similarly,

$$\frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right) \left(\frac{\dot{B}}{B} + \frac{\dot{F}}{F} \right) = 0 \quad (17)$$

IV. ISOTROPY AND ALLOWED SOLUTIONS

The deceleration parameter is defined by

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 \quad (18)$$

We assume the special form

$$q(t) = -1 + (q_0 + 1)e^{-\alpha t} \quad (19)$$

Let $\beta = q_0 + 1$, so that $q(t) = -1 + \beta e^{-\alpha t}$. Integrating (18) gives

$$a(t) = (e^{\alpha t} - 1)^{\frac{1}{\beta}} \quad (20)$$

We also assume the power-law form used in [4],

$$F \propto a^n \quad (21)$$

Thus

$$F = r(e^{\alpha t} - 1)^{\frac{n}{\beta}} \quad (22)$$

Set $X = e^{\alpha t} - 1$. Then $A = X^{p_1}$, $B = X^{p_2}$, $C = X^{p_3}$, $F = X^{p_F}$, where $p_1 = \frac{1}{\beta}$, $p_F = \frac{n}{\beta}$. Substituting into (16) gives

$$(p_2 - p_1) \left[\frac{\alpha(X+1)}{X} \right] \left[\alpha + (p_1 + p_2 + p_3 + p_F - 1) \frac{\alpha(X+1)}{X} \right] = 0 \quad (23)$$

Since $\frac{\alpha(X+1)}{X}$ is not constant for $\alpha \neq 0$, we must have $p_2 = p_1$. Similarly, (17) gives $p_3 = p_1$. Therefore $A = B = C = a$. Thus the field equations force the model to be isotropic. In particular, the earlier assumption $B = C^d$ leads to $d = 1$. Consequently, the anisotropic Bianchi type-V solution with $d \neq 1$ is not admissible.

For the isotropic case, $A = B = C = a = X^{\frac{1}{\beta}}$. The field equation (13) reduces to

$$\ddot{F} - H\dot{F} + 2F\dot{H} + \frac{2b^2}{a^2}F + k(\rho + p) = 0 \quad (24)$$

where $H = \dot{a}/a$. Using $\rho = c_2 X^{-\frac{3(1+\omega)}{\beta}}$, and substituting $F = rX^{\frac{n}{\beta}}$, $a = X^{\frac{1}{\beta}}$, equation (24) gives

$$-\frac{(n+2)\alpha^2}{\beta}(X+1) + \frac{n(n-1)\alpha^2}{\beta^2}(X+1)^2 + 2b^2X^{2-\frac{2}{\beta}} + \frac{k(1+\omega)c_2}{r}X^{2-\frac{n+3(1+\omega)}{\beta}} = 0 \quad (25)$$

For this to hold for all t , the exponent of the b -term must be 0, 1, or 2. Since $b \neq 0$, this requires $\beta = 1$ or $\beta = 2$. The case $\beta = 1$ gives no physically admissible solution with $0 \leq \omega \leq 1$. The only viable case is $\beta = 2$. Then (25) becomes

$$-\frac{(n+2)\alpha^2}{2}(X+1) + \frac{n(n-1)\alpha^2}{4}(X+1)^2 + 2b^2X + \frac{k(1+\omega)c_2}{r}X^{2-\frac{n+3(1+\omega)}{2}} = 0 \quad (26)$$

The density exponent must be 0, 1, or 2. The only physical choices are:

Case 1: Dust.

$$\omega = 0, \quad n = 1, \quad \alpha^2 = \frac{4b^2}{3}, \quad r = \frac{kc_2}{2b^2}$$

Case 2: Radiation.

$$\omega = \frac{1}{3}, \quad n = 0, \quad \alpha^2 = 2b^2, \quad r = \frac{2kc_2}{3b^2}$$

Thus $q_0 = 1$, and the deceleration parameter is

$$q(t) = -1 + 2e^{-\alpha t}$$

V. PHYSICAL PARAMETERS AND DISCUSSION

For the allowed isotropic model,

$$ds^2 = dt^2 - X[dx^2 + e^{2bx}(dy^2 + dz^2)], \quad X = e^{\alpha t} - 1 \quad (27)$$

The physical parameters are

$$V = X^{3/2} \quad (28)$$

$$H = \frac{\alpha(X+1)}{2X} \quad (29)$$

$$\theta = \frac{3\alpha(X+1)}{2X} \quad (30)$$

$$A_m = 0 \quad (31)$$

$$\sigma^2 = 0 \quad (32)$$

$$q = -1 + 2e^{-\alpha t} \quad (33)$$

The Ricci scalar becomes

$$R = -\frac{3\alpha^2(X+1)}{X} + \frac{6b^2}{X} \quad (34)$$

For the dust case,

$$R = \frac{2b^2(1-2X)}{X}$$

For the radiation case,

$$R = -6b^2$$

The function $f(R)$ is obtained from

$$f(R) = \frac{FR}{2} + \frac{3}{2}(\ddot{F} + 3H\dot{F}) - \frac{k}{2}(1-3\omega)\rho \quad (35)$$

For the dust case, $F = rX^{1/2}$, $\rho = c_2X^{-3/2}$, and

$$f(R) = 4rb^2X^{-1/2} \quad (36)$$

Using $X = 2b^2/(R + 4b^2)$, this gives

$$f(R) = \frac{2\sqrt{2}r|b|}{\sqrt{R+4b^2}} \quad (37)$$

For the radiation case, $F = r$, $\omega = 1/3$, and $T = \rho - 3p = 0$. Hence

$$f(R) = \frac{r}{2} R \quad (38)$$

The model has an initial singularity at $t = 0$, where $X = 0$, $a(0) = 0$, and $H \rightarrow \infty$. At $t = 0$, $q(0) = 1$, so the universe initially decelerates. The deceleration parameter vanishes at $t = (\ln 2)/\alpha$, and as $t \rightarrow \infty$, $q \rightarrow -1$. Thus the late-time universe is in an accelerating de Sitter-like phase. The correct field equations do not admit the previously claimed anisotropic $d \neq 1$ Bianchi type-V solution; they force isotropy. Only the dust and radiation perfect-fluid cases above are compatible with the assumed deceleration parameter and power-law $F \propto a^n$.

VI. CONCLUSION

In this work, the Bianchi type-V perfect-fluid cosmological model has been re-examined within the framework of $f(R)$ theory of gravity, under the special deceleration parameter $q(t) = -1 + (q_0 + 1)e^{-\alpha t}$ and the power-law ansatz $F \propto a^n$. Careful use of the correct diagonal field equations shows that these two assumptions together are consistent only with a fully isotropic expansion, $A = B = C = a$; the anisotropic Bianchi type-V configuration with $B = C^d$, $d \neq 1$, previously reported in the literature, is not admissible. Enforcing consistency of the field equations for all cosmic time further restricts the free parameters to $\beta = q_0 + 1 = 2$, so that $q_0 = 1$, leaving exactly two physically viable perfect-fluid solutions: a dust-filled universe ($\omega = 0$, $n = 1$, $\alpha^2 = 4b^2/3$) and a radiation-filled universe ($\omega = 1/3$, $n = 0$, $\alpha^2 = 2b^2$).

For both solutions the model begins with a point-type initial singularity at $t = 0$, where the volume, Hubble parameter, and expansion scalar diverge, while the shear scalar and anisotropy parameter vanish identically ($\sigma^2 = 0$, $A_m = 0$) for all time, confirming that the model is shear-free and isotropic throughout its evolution. The deceleration parameter evolves from $q(0) = 1$ (initial deceleration) through $q = 0$ at $t = (\ln 2)/\alpha$ to $q \rightarrow -1$ as $t \rightarrow \infty$, so the model naturally interpolates between an early decelerating phase and a late-time accelerating, de-Sitter-like phase, in qualitative agreement with current cosmological observations. The Ricci scalar and the reconstructed $f(R)$ differ in character between the two cases: the dust model yields a Ricci scalar that depends on cosmic time and an $f(R)$ of the form $f(R) \propto (R + 4b^2)^{-1/2}$, whereas the radiation model yields a constant Ricci scalar $R = -6b^2$ together with the standard linear form $f(R) = \frac{r}{2}R$, consistent with the traceless character of radiation. Overall, the analysis demonstrates that once the field equations are treated correctly, the assumed deceleration-parameter and power-law forms are compatible only with an isotropic, singularity-generating cosmology of dust or radiation type, providing a consistent and physically admissible extension of earlier Bianchi type-V studies in $f(R)$ gravity. Future work may extend this analysis to bulk-viscous and scalar-field matter sources, and to a numerical comparison of the derived $f(R)$ forms against observational constraints on the growth of large-scale structure.

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